First Derivative Test (p. 209)
First Derivative Test for Local Extrema (p. 209)
geometric mean (p. 208)
global maximum value (p. 187)
global minimum value (p. 187)
increasing function (p. 202)
linear approximation (p. 237)
linearization (p. 237)
local linearity (p. 237)
local maximum value (p. 193)
local minimum value (p. 193)
logistic or life cycle curve (p. 214)
logistic regression (p. 215)
marginal analysis (p. 227)
marginal cost and revenue (p. 227)
Mean Value Theorem (p. 200)
monotonic function (p. 208)
Newton's method (p. 244)
optimization (p. 223)
percentage change (p. 242)
point of inflection (p. 212)
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relative extrema (p. 193)
Rolle's Theorem (p. 200)
Second Derivative Test for Local Extrema (p. 215)
standard linear approximation (p. 237)
stationary point (p. 194)

Chapter 5 Review Exercises

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

The collection of exercises marked in red could be used as a chapter test.

In Exercises 1 and 2, use analytic methods to find the global extreme values of the function on the interval and state where they occur.

1. $y = x\sqrt{2 - x}, \quad -2 \le x \le 2$

2. $y = x^3 - 9x^2 - 21x - 11, \; -\infty < x < \infty$

In Exercises 3 and 4, use analytic methods. Find the intervals on which the function is

(a) increasing, (b) decreasing,
(c) concave up, (d) concave down.

Then find any

(e) local extreme values, (f) inflection points.

3. $y = x^2 e^{1/x^2}$ **4.** $y = x\sqrt{4 - x^2}$

In Exercises 5–16, use analytic methods to find the intervals on which the function is

(a) increasing, (b) decreasing,
(c) concave up, (d) concave down.

Support your answers graphically. Then find any

(e) local extreme values, (f) inflection points.

5. $y = 1 + x - x^2 - x^4$ **6.** $y = e^{x-1} - x$

7. $y = \dfrac{1}{\sqrt[4]{1 - x^2}}$. **8.** $y = \dfrac{x}{x^3 - 1}$

9. $y = \cos^{-1} x$ **10.** $y = \dfrac{x}{x^2 + 2x + 3}$

11. $y = \ln|x|, -2 \le x \le 2, \quad x \ne 0$

12. $y = \sin 3x + \cos 4x, \quad 0 \le x \le 2\pi$

13. $y = \begin{cases} e^{-x}, & x \le 0 \\ 4x - x^3, & x > 0 \end{cases}$

14. $y = -x^5 + \frac{7}{3}x^3 + 5x^2 + 4x + 2$

15. $y = x^{4/5}(2 - x)$

16. $y = \dfrac{5 - 4x + 4x^2 - x^3}{x - 2}$

In Exercises 17 and 18, use the derivative of the function $y = f(x)$ to find the points at which f has a

(a) local maximum, (b) local minimum, or
(c) point of inflection.

17. $y' = 6(x + 1)(x - 2)^2$ **18.** $y' = 6(x + 1)(x - 2)$

In Exercises 19–22, find all possible functions with the given derivative.

19. $f'(x) = x^{-5} + e^{-x}$ **20.** $f'(x) = \sec x \tan x$

21. $f'(x) = \dfrac{2}{x} + x^2 + 1, \; x > 0$ **22.** $f'(x) = \sqrt{x} + \dfrac{1}{\sqrt{x}}$

In Exercises 23 and 24, find the function with the given derivative whose graph passes through the point P.

23. $f'(x) = \sin x + \cos x, \quad P(\pi, 3)$

24. $f'(x) = x^{1/3} + x^2 + x + 1, \quad P(1, 0)$

In Exercises 25 and 26, the velocity v or acceleration a of a particle is given. Find the particle's position s at time t.

25. $v = 9.8t + 5, \; s = 10$ when $t = 0$

26. $a = 32, \; v = 20$ and $s = 5$ when $t = 0$

In Exercises 27–30, find the linearization $L(x)$ of $f(x)$ at $x = a$.

27. $f(x) = \tan x, \quad a = -\pi/4$ **28.** $f(x) = \sec x, \quad a = \pi/4$

29. $f(x) = \dfrac{1}{1 + \tan x}, \; a = 0$ **30.** $f(x) = e^x + \sin x, \; a = 0$

In Exercises 31–34, use the graph to answer the questions.

31. Identify any global extreme values of f and the values of x at which they occur.

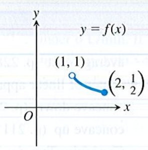


Figure for Exercise 31

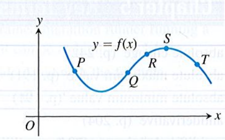


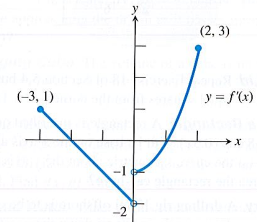
Figure for Exercise 32

32. At which of the five points on the graph of $y = f(x)$ shown here

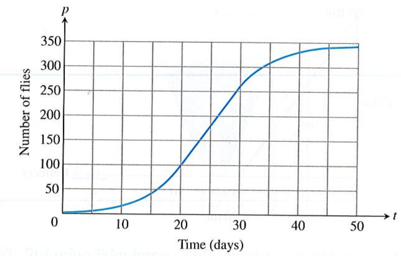
(a) are y' and y'' both negative?

(b) is y' negative and y'' positive?

33. Estimate the intervals on which the function $y = f(x)$ is (a) increasing; (b) decreasing. (c) Estimate any local extreme values of the function and where they occur.



34. Here is the graph of the fruit fly population from Section 2.4, Example 2. On approximately what day did the population's growth rate change from increasing to decreasing?



35. ***Connecting f and f′*** The graph of f' is shown in Exercise 33. Sketch a possible graph of f given that it is continuous with domain $[-3, 2]$ and $f(-3) = 0$.

36. ***Connecting f, f′, and f″*** The function f is continuous on $[0, 3]$ and satisfies the following.

x	0	1	2	3
f	0	−2	0	3
f'	−3	0	does not exist	4
f''	0	1	does not exist	0

x	$0 < x < 1$	$1 < x < 2$	$2 < x < 3$
f	−	−	+
f'	−	+	+
f''	+	+	+

(a) Find the absolute extrema of f and where they occur.

(b) Find any points of inflection.

(c) Sketch a possible graph of f.

37. ***Mean Value Theorem*** Let $f(x) = x \ln x$.

(a) **Writing to Learn** Show that f satisfies the hypotheses of the Mean Value Theorem on the interval $[a, b] = [0.5, 3]$.

(b) Find the value(s) of c in (a, b) for which

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

(c) Write an equation for the secant line AB where $A = (a, f(a))$ and $B = (b, f(b))$.

(d) Write an equation for the tangent line that is parallel to the secant line AB.

38. ***Motion along a Line*** A particle is moving along a line with position function $s(t) = 3 + 4t - 3t^2 - t^3$. Find the (a) velocity and (b) acceleration, and (c) describe the motion of the particle for $t \ge 0$.

39. ***Approximating Functions*** Let f be a function with $f'(x) = \sin x^2$ and $f(0) = -1$.

(a) Find the linearization of f at $x = 0$.

(b) Approximate the value of f at $x = 0.1$.

(c) **Writing to Learn** Is the actual value of f at $x = 0.1$ greater than or less than the approximation in (b)?

40. ***Differentials*** Let $y = x^2 e^{-x}$. Find (a) dy and (b) evaluate dy for $x = 1$ and $dx = 0.01$.

41. Table 5.5 shows the growth of the U.S. population from 1790 to 2000 in 30-year increments and includes the growth in 2010.

TABLE 5.5 U.S. Population Growth

Year	Growth
1790	3929
1820	9638
1850	23,192
1880	50,189
1910	92,228
1940	132,165
1970	203,302
2000	281,422
2010	309,447

Source: http://www.census.gov/population/censusdata/table-4.pdf US Census Bureau; (2010) http://www.census.gov/

(a) Find the logistic regression for the data.

(b) Graph the data in a scatter plot and superimpose the regression curve.

(c) Use the regression equation to predict the U.S. population in 2050.

(d) About what year is the U.S. population growing the fastest? What significant behavior does the graph of the regression equation exhibit at that point?

(e) What does the regression equation indicate about the U.S. population in the long run?

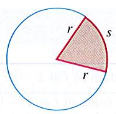
(f) **Writing to Learn** Is your answer to part (e) reasonable?

42. *Newton's Method* Use Newton's method to estimate all real solutions to $2\cos x - \sqrt{1+x} = 0$. State your answers accurate to 6 decimal places.

43. *Rocket Launch* A rocket lifts off the surface of Earth with a constant acceleration of 20 m/sec^2. How fast will the rocket be going 1 min later?

44. *Launching on Mars* The acceleration of gravity near the surface of Mars is 3.72 m/sec^2. If a rock is blasted straight up from the surface with an initial velocity of 93 m/sec (about 208 mph), how high does it go?

45. *Area of Sector* If the perimeter of the circular sector shown here is fixed at 100 ft, what values of r and s will give the sector the greatest area?

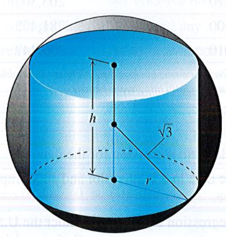


46. *Area of Triangle* An isosceles triangle has its vertex at the origin and its base parallel to the x-axis with the vertices above the axis on the curve $y = 27 - x^2$. Find the largest area the triangle can have.

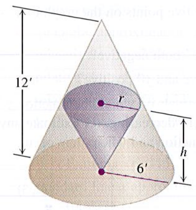
47. *Storage Bin* Find the dimensions of the largest open-top storage bin with a square base and vertical sides that can be made from 108 ft^2 of sheet steel. (Neglect the thickness of the steel and assume that there is no waste.)

48. *Designing a Vat* You are to design an open-top rectangular stainless-steel vat. It is to have a square base and a volume of 32 ft^3, to be welded from quarter-inch plate, and weigh no more than necessary. What dimensions do you recommend?

49. *Inscribing a Cylinder* Find the height and radius of the largest right circular cylinder that can be put into a sphere of radius $\sqrt{3}$, as described in the figure.



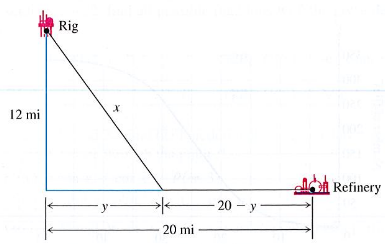
50. *Cone in a Cone* The figure shows two right circular cones, one upside down inside the other. The two bases are parallel, and the vertex of the smaller cone lies at the center of the larger cone's base. What values of r and h will give the smaller cone the largest possible volume?



51. *Box with Lid* Repeat Exercise 18 of Section 5.4 but this time remove the two equal squares from the corners of a 15-in. side.

52. *Inscribing a Rectangle* A rectangle is inscribed under one arch of $y = 8\cos(0.3x)$ with its base on the x-axis and its upper two vertices on the curve symmetric about the y-axis. What is the largest area the rectangle can have?

53. *Oil Refinery* A drilling rig 12 mi offshore is to be connected by a pipe to a refinery onshore, 20 mi down the coast from the rig, as shown in the figure. If underwater pipe costs \$40,000 per mile and land-based pipe costs \$30,000 per mile, what values of x and y give the least expensive connection?



54. *Designing an Athletic Field* An athletic field is to be built in the shape of a rectangle x units long capped by semicircular regions of radius r at the two ends. The field is to be bounded by a 400-m running track. What values of x and r will give the rectangle the largest possible area?

55. *Manufacturing Tires* Your company can manufacture x hundred grade A tires and y hundred grade B tires a day, where $0 \le x \le 4$ and

$$y = \frac{40 - 10x}{5 - x}.$$

Your profit on a grade A tire is twice your profit on a grade B tire. What is the most profitable number of each kind to make?

56. *Particle Motion* The positions of two particles on the s-axis are $s_1 = \cos t$ and $s_2 = \cos(t + \pi/4)$.

(a) What is the farthest apart the particles ever get?

(b) When do the particles collide?

57. *Open-top Box* An open-top rectangular box is constructed from a 10-by 16-in. piece of cardboard by cutting squares of equal side length from the corners and folding up the sides. Find analytically the dimensions of the box of largest volume and the maximum volume. Support your answers graphically.

58. *Changing Area* The radius of a circle is changing at the rate of $-2/\pi$ m/sec. At what rate is the circle's area changing when $r = 10$ m?

59. *Particle Motion* The coordinates of a particle moving in the plane are differentiable functions of time t with $dx/dt = -1$ m/sec and $dy/dt = -5$ m/sec. How fast is the particle approaching the origin as it passes through the point $(5, 12)$?

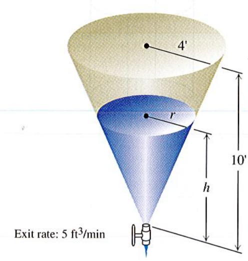
60. *Changing Cube* The volume of a cube is increasing at the rate of 1200 cm^3/min at the instant its edges are 20 cm long. At what rate are the edges changing at that instant?

61. *Particle Motion* A point moves smoothly along the curve $y = x^{3/2}$ in the first quadrant in such a way that its distance from the origin increases at the constant rate of 11 units per second. Find dx/dt when $x = 3$.

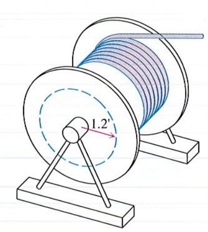
62. *Draining Water* Water drains from the conical tank shown in the figure at the rate of 5 ft^3/min.

(a) What is the relation between the variables h and r?

(b) How fast is the water level dropping when $h = 6$ ft?

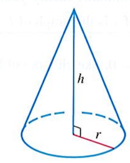


63. *Stringing Telephone Cable* As telephone cable is pulled from a large spool to be strung from the telephone poles along a street, it unwinds from the spool in layers of constant radius, as suggested in the figure. If the truck pulling the cable moves at a constant rate of 6 ft/sec, use the equation $s = r\theta$ to find how fast (in rad/sec) the spool is turning when the layer of radius 1.2 ft is being unwound.



64. *Throwing Dirt* You sling a shovelful of dirt up from the bottom of a 17-ft hole with an initial velocity of 32 ft/sec. Is that enough speed to get the dirt out of the hole, or had you better duck?

65. *Estimating Change* Write a formula that estimates the change that occurs in the volume of a right circular cone (see figure) when the radius changes from a to $a + dr$ and the height does not change.



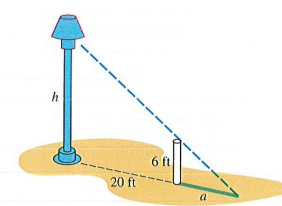
66. *Controlling Error*

(a) How accurately should you measure the edge of a cube to be reasonably sure of calculating the cube's surface area with an error of no more than 2%?

(b) Suppose the edge is measured with the accuracy required in part (a). About how accurately can the cube's volume be calculated from the edge measurement? To find out, estimate the percentage error in the volume calculation that might result from using the edge measurement.

67. *Compounding Error* The circumference of the equator of a sphere is measured as 10 cm with a possible error of 0.4 cm. This measurement is used to calculate the radius. The radius is then used to calculate the surface area and volume of the sphere. Estimate the percentage errors in the calculated values of **(a)** the radius, **(b)** the surface area, and **(c)** the volume.

68. *Finding Height* To find the height of a lamppost (see figure), you stand a 6-ft pole 20 ft from the lamp and measure the length a of its shadow, finding it to be 15 ft, give or take an inch. Calculate the height of the lamppost using the value $a = 15$, and estimate the possible error in the result.



69. *Decreasing Function* Show that the function $y = \sin^2 x - 3x$ decreases on every interval in its domain.